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BOOK of ABSTRACTS

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Dynamics of a spin-transfer nanooscillator controlled by external magnetic field.

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Spin-transfer nanooscillators (STNO) are promising nanoscale devices for applications in modern high-tech electronics and could be exploited in a different ways, for instance: as extremely small ultra-high frequency generators with central frequency from 10 to 100 GHz and beyond; as a trivial memory cell with two states and much more. In current work we study dynamical regimes of STNO with nanopillar unidomain structure under external magnetic field impact and their mutual transitions controlled by variations of control parameters. One can find that with a certain values of control parameters the STNO could function in autooscillatory regime; in stable state with constant magnetization; in multistable states of different types: stable/stable and stable/autooscillatory.

Chimera state realization in chaotic systems. The role of hyperbolicity

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In the present talk the transition from spatial coherence regimes to a noncoherence regime is analyzed in ensembles of discrete and continuous chaotic systems with nonlocal couplings. The possibility of existence of so-called "chimera" states is studied for a ring of nonlocally coupled identical chaotic oscillators. We consider nonhyperbolic systems which include maps with a single quadratic extremum, differential systems with Shil'nikov's type attractor as well as a class of hyperbolic systems which enclose the Lozi map and the Lorenz system. We hypothesize that two basic models, i.e., the Henon map and the Lozi map, can be used to generalize the implementation of chimera states to a sufficiently wide calss of chaotic systems. The Henon map can be served as an example of a partial system to describe the realization of chimera states in a ring of one-dimensional Feigenbaum's type maps, two-dimensional maps with period-doublings as well as differential systems with a saddle-focus separatrix loop according to Shil'nikov. The Lozi map can model the dynamics of ensembles of Lorenz-type oscillators. Our hypothesis consists in that the chimera states can be observed in ensembles of nonhyperbolic oscillators which can typically demonstrate the effect of multistability. The chimera modes cannot exist in ensembles whose partial element is represented by a hyperbolic system. Our suggestion is confirmed by the results of the previously published works.

In the talk we describe the numerical simulation results for the transition "hyperbolicity – nonhyperbolicity" in a ring of nonlocally coupled Lorenz oscillators. It is shown that chimera states cannot be realized in the hyperbolic regime. However, if the parameters of a partial Lorenz system are chosen in the nonhyperbolic region, the chimera states can be confidently observed. These results can be considered as a further confirmation of the proposed hyporthesis.

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Singularly Hyperbolic Attractors in Phase Systems

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An attractor is singularly hyperbolic if it is hyperbolic in terms of invariant cones' existence at each point of a versal parameter interval and the structure of the attractor changes at a set of bifurcational points within this parameter interval.

In this talk, we consider two types of phase systems in discrete and continuous type and prove the existence of singularly hyperbolic attractors. The first, discrete-time system is

$$\begin{aligned} &x(i+1) = Ax(i) + Bf(\varphi(i)), \\ &\varphi = C^T x, \end{aligned} \tag{1}$$

where $x(i) \in R^n$, $f(\varphi(i))$ is a 2π-periodic scalar function, A, B, C are constant matrices. We use the comparison method to derive the conditions for the existence of a singular hyperbolic attractor in this system.

The second, continuous-time system, describing the interaction between oscillators of different types, reads

$$\begin{aligned} &\ddot{\varphi} + f(\varphi, \dot{\varphi})\dot{\varphi} + g(\varphi) = -\mu \ddot{y} \\ &\ddot{y} + h\dot{y} + y = -\mu \ddot{\varphi}, \end{aligned} \tag{2}$$

where $y \in R^1$, $\varphi \in S^1$, f and g are 2π-periodic functions. A representative example of this general phase system is oscillators connected via the Huygens's coupling [1,2]. For the general system (2) as well as for the Huygens-type system [1], we prove the existence of a wild attractor which belongs to the class of singularly hyperbolic attractors and has both oscillatory and rotatory trajectories.

This work was supported by the Russian Science Foundation under the grant "Phase dynamics of oscillatory media" and by the Russian Foundation for Basic Research (grants No 12-01-00694 and 14-02-31727).

[1] V.N. Belykh and E.V. Pankratova, 'Shilnikov chaos in oscillators with Huygens coupling," Int. J. Bifurcation and Chaos, vol. 24, 1440007, 2014.

[2] E.V. Pankratova and V.N. Belykh, "Consequential noise-induced synchronization of indirectly coupled self-sustained oscillators," Eur. Phys. J. Special Topics, vol. 222, pp. 2509–2515, 2013.

Repulsive Inhibition Promotes Synchrony in Excitatory Neurons: Help from the Enemy

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We show that the addition of pairwise repulsive inhibition to excitatory networks of bursting neurons induces synchrony, in contrast to one's expectations [1,2]. Through stability analysis, we reveal the mechanism of this purely synergistic phenomenon and demonstrate that the inhibition leads to the disappearance of a homoclinic bifurcation that governs the type of synchronized bursting. As a result, the inhibition causes the transition from square-wave to easier-to-synchronize plateau bursting, so that weaker excitation is sufficient to induce bursting synchrony. This effect is generic and observed in different models of bursting neurons and fast synaptic interactions. We also find a universal scaling law for the synchronization stability condition for large networks in terms of the number of excitatory and inhibitory inputs each neuron receives, regardless of the network size and topology.

We dedicate this talk to the memory of Leonid P. Shilnikov, the pioneer of homoclinic bifurcation theory, and emphasize the importance of homoclinic bifurcations for understanding the onset of synchronization in bursting networks.

This work was supported by the National Science Foundation under Grant No. DMS-1009744, the US ARO Network Sciences Program, and GSU Brains & Behavior program.

[1]I. Belykh, R. Reimbayev and K. Zhao, "Synergistic effect of repulsive inhibition in synchronization of excitatory networks," Phys. Rev. E, 2015 (to appear).

[2]R. Reimbayev and I. Belykh, "When transitions between bursting modes induce neural synchrony," Int. J. Bifurcation and Chaos, vol. 24, 1440013, 2014.

Invariants of the Orthodox Paschalion

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Hierarchy of dynamics and the Hamiltonization of nonholonomic systems

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The destruction of conservative dynamics in the system of phase equations by symmetry breaking

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It is well known [1,2] that if the equations for reversible dissipative flow system have a specific symmetry (i.e. they are invariant relatively to a coordinate transformation), the conservative dynamics can appear in this system. For example, this phenomenon occurs in the system of phase equations for a chain of coupled oscillators [3]:

$$\dot{\psi}_k = \Delta_k + \varepsilon f(\psi_{k-1} + \varepsilon f(\psi_{k+1} - 2\varepsilon f(\psi_k),$$

where ψ_k is the difference of phases of neighbor oscillators and $f(\psi)$ is the coupling function. In the case of four oscillators the system is invariant relatively to the involution $\psi_k \to \pi - \psi_{n-k}$ if coupling function contains only odd Fourier harmonics, e.g. $f(\psi) = \sin\psi + A\sin 3\psi$ as it was taken in [3].

We study the effect of symmetry breaking on the dynamics of this system. We consider $f(\psi) = \sin\psi + (A-d)\sin 3\psi + d\sin 2\psi$ as the coupling function so d is the symmetry breaking parameter.

To investigate the system we obtained numerically the Poincare map with the symmetric plane $\psi_2 = \pi/2$ as a section plane. We reveal that the stable cycles of different periods appear with the increase of the parameter d but most of them exist in narrow band of parameter d values. We obtain the stable and unstable manifolds of saddle cycles and show that homo- and heteroclinic structures exist.

Also we reveal that the quasistable set of complex structure appears in some band of parameter d values. The point moves along this set during a thousands of iterations but finally comes to the stable cycle.

This work was supported by RFBR (project 14-02-31064) and the presidential program for support of leading scientific schools NSh-1726.2014.2.

[1] Lamb, J.S. W. and Stenkin, O.V. //Nonlinearity, 2004, vol.17, no.4, p. 1217.
[2] Delshams, A. et al. //Nonlinearity, 2013, vol.26, no.1, p. 1
[3] D. Topaj, A. Pikovsky. //Physica D, 170 (2002), p.118

A study of the stochastic resonance as a random dynamical system

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We study a standard model for the stochastic resonance from the point of view of dynamical systems. We present a framework for random dynamical systems with nonautonomous deterministic forcing and we prove the existence of an attracting random periodic orbit for a class of one-dimensional systems with a time-periodic component. In the case of the stochastic resonance, we can derive an indicator for the resonant regime.

The Lorenz system near the loss of the foliation condition

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The well-known Lorenz system is classically studied via its reduction to the one-dimensional Lorenz map, which captures the full behaviour of the dynamics of the system. The reduction requires that the stable and unstable foliations on the classic Poincaré section are transverse locally near the chaotic attractor. We study a parameter regime where this so-called foliation condition fails for the first time and, subsequently, the Lorenz map no longer accurately represents the dynamics. Specifically, we investigate the development of hooks in the Poincaré return map that marks the loss of the foliation condition. To this end, we study how the three-dimensional phase space is organised by the global invariant manifolds of saddle equilibria and saddle periodic orbits, where we make extensive use of the continuation of orbit segments formulated by a suitable two-point boundary value problem (BVP). In particular, we compute the intersection curves of the two-dimensional unstable manifold $W^u(\Gamma)$ of a periodic orbit Γ with the Poincaré section. We identify when hooks form in the Poincaré map by formulating as a BVP the point of tangency between $W^u(\Gamma)$ and the stable foliation. This approach allows us to continue this tangency accurately and efficiently in each pair of the three system parameters. In this way, we identify the conic region in parameter space where the classical Lorenz attractor exists. As is expected from earlier work by Bykov and Shilnikov, a curve of T-points lies on the bounding surface, and we show that it ends in a codimension-three T-point-Andronov-Hopf bifurcation point.

Title Diffusion through non-transverse heteroclinic chains: A long-time instability for the NLS

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We introduce a new mechanism for instability (diffusion) in dynamical systems, based in the shadowing of a sequence of invariant tori connected along *non-transverse* heteroclinic orbits, under some geometric restrictions. This mechanism can be readily applied to systems of large dimensions, like infinite-dimensional Hamiltonian systems, particularly the Nonlinear Schrödinger Equation with cubic defocusing.

This is a joint work with Adrià Simon and Piotr Zgliczcynski.

Strange Nonchaotic Attractor of Hunt and Ott Type in a System with Ring Geometry

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The objective of this report is to introduce a physically realizable system with robust strange nonchaotic attractor (SNA).

Hunt and Ott offered an artificial system in which presence of the robust SNA is proven [1]. An example of some real system was suggested by Kuznetsov and Jalnine [2]. In contrast to the work of Kuznetsov and Jalnine, where the scheme was composed of self-oscillating elements, the system presented here is a ring system of two damping oscillators with additional elements introducing amplification and nonlinearity. The model is similar to that suggested in Ref. [3], but differs in the phase transition rules between the alternately exciting oscillatory subsystems.

Let the natural frequency of the first oscillator be ω_0 and that of the second oscillator be $2\omega_0$. The model equations of the system are:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \varepsilon\frac{d}{dt}y\sin x(\omega_0 t + \theta), \tag{3}$$

$$\ddot{y} + \gamma\dot{y} + 4\omega_0^2 y = \varepsilon\frac{d}{dt}\frac{x}{\sqrt{1+x^2}}g(t), \tag{4}$$

$$\dot{\theta} = \frac{2\pi\omega}{T}, \omega = \frac{\sqrt{5}-1}{2}. \tag{5}$$

Here x and y are the generalized coordinates of the oscillatory subsystems, γ is a damping factor, ε is a small parameter, α is an amplification factor. The function $g(t)$ describes the external signal, being switched on for short time intervals τ with the period $\frac{2\pi N}{\omega_0}$, where N is integer:

$$g(t) = \begin{cases} a^2, & nT \le t < \tau, \\ 0, & nT + \tau \le t < (n+1)T. \end{cases} \tag{6}$$

Numerical simulation confirms that the evolution of the first oscillator phase is described by the map of Hant and Ott type. Some results of are presented in Figure 1. Figure 1a shows the phase portrait of the system in the three-dimensional state space. Figure 1b illustrates the basic topological characteristic of the phase transfer. The vertical axis corresponds to phase of the first oscillator and the horizontal axis to the phase θ of external force with incommensurate frequency ω. The curve C_1 which turns around the torus in the θ direction is transformed by the map effect to the curve C_2, which makes one turn around the meridian and one turn around the parallel of the torus. The form of these curves just corresponds to topological nature of the map for the Hunt and Ott model.

Behavior of the system also is illustrated by Fourier spectrum shown in Figure 1c.

The dynamics of this system was explored in a quite wide range of parameters and manifests robustness analogous to that of the artificial model of Hunt and Ott.

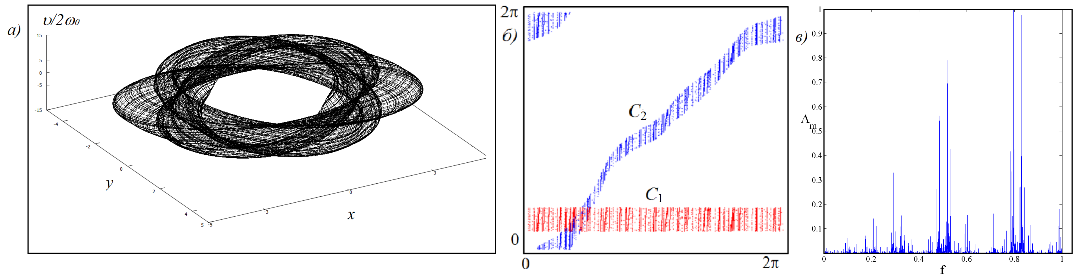

Figure 1. Phase portrait of the system in three-dimensional space (a). Numerical illustration of the basic topological properties of the phase transfer on the plot of φ versus θ (b). Fourier spectrum of signal produced by the system (c). Parameters: $\omega_0 = 6\pi, \tau = 3, \gamma = 0.25, a = 5$.

1. B.R. Hunt and E. Ott. Fractal Properties of Robust Strange Nonchaotic Attractors // *Phys. Rev. Lett.* 2001. Vol. 87. P. 254101.
2. A.Yu.Jalnine and S.P.Kuznetsov. On the Realization of the Hunt–Ott Strange Nonchaotic Attractor in a Physical System // *Technical Physics*. 2007. Vol. 52. No 4. P. 401–408.
3. V. P. Kruglov. Ring nonautonomous generator of hyperbolic chaos // *Izvestiya VUZ. Applied Nonlinear Dynamics*. 2010. Vol. 18. No 5. P. 138-150.

Concept of Stability as a Whole of a Family of Fibers Maps for C^1-Smooth Skew Products and Its Generalization

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We consider here the concepts of stability as a whole and introduce the concept of dense stability as a whole of a family of fibers maps for C^1-smooth skew products of maps of an interval with a complicated dynamics of a quotient map.

We prove the criterion of Ω-stability based on the concept of stability as a whole of a family of fibers maps for C^1-smooth skew products of maps of an interval. Also we prove the existence theorems for C^1-smooth skew products with densely stable as a whole families of fibers maps.

Then, we investigate approximate properties of C^1-smooth skew products of maps of an interval with mentioned above properties of families of fibers maps.

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Variety of strange pseudohyperbolic attractors in three-dimensional generalized Hénon maps

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We propose a sufficiently simple method, *the method of saddle charts*, for finding homoclinic attractors (here, strange attractors with saddle fixed points). This method is based on a preliminary detection of parameter domains related to the existence of fixed points with a given set of multipliers, e.g. such that a potential strange attractor with this point can be of Lorenz type. The obtained conditions are, in fact, only necessary ones, however, they simplify essentially a procedure of finding homoclinic attractors. In the paper, we apply the method of saddle charts for three-dimensional generalized Hénon maps for which we find "a zoo" of homoclinic attractors of various types.

Reversible mixed dynamics

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We say that a system possesses a mixed dynamics if 1) it has infinitely many hyperbolic periodic orbits of all possible types (stable, unstable,saddle) and 2) the closures of the sets of orbits of different types have nonempty intersections. Recall that Newhouse regions are open domains (from the space of smooth dynamical systems) in which systems with homoclinic tangencies are dense. Newhouse regions in which systems with mixed dynamics are generic (compose residual subsets) are called absolute Newhouse regions. Their existence was proved in the paper [1] for the case of 2d diffeomorphisms close to a diffeomorphism with a nontransversal heteroclinic cycle containing two fixed (periodic) points with the Jacobians less and greater than 1. Fundamentally that "mixed dynamics"is the universal property of reversible chaotic systems. Moreover, in this case generic systems from absolute Newhouse regions have infinitely many stable, unstable, saddle and elliptic periodic orbits [2,3].

As well-known, reversible systems are often met in applications and they can demonstrate a chaotic orbit behavior. However, the phenomenon of mixed dynamics means that this chaos can not be associated with "strange attractor"or "conservative chaos". Attractors and repellers have here a nonempty intersection containing symmetric orbits (elliptic and saddle ones) but do not coincide, since periodic sinks (sources) do not belong to the repeller (attractor). Therefore, "mixed dynamics"should be considered as a new form of dynamical chaos posed between "strange attractor"and "conservative chaos".

These and related questions will be discussed in the talk.

[1] S.Gonchenko, D.Turaev, L.Shilnikov *Proc. Steklov Inst. Math.*, 1997, **216**, 70-118.
[2] J.S.W.Lamb, O.V.Stenkin *Nonlinearity*, 2004, v.17(4), 1217-1244.
[3] A.Delshams *et al Nonlinearity*, 2013, v.26(1), 1-33.

Homoclinic tangencies in area-preserving and orientation-reversing maps

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We study bifurcations of area-preserving and orientation-reversing maps with quadratic homoclinic tangencies. We study the case when the maps are given on non-orientable two-dimensional surfaces as well as the case of maps with non-orientable saddles. We consider one and two parameter general unfoldings and establish the results related to the emergence of elliptic periodic orbits. This is a joint work with A. Delshams and S. Gonchenko.

On interrelation between dynamics and topology of ambient manifold

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Our lectures are devoted to Morse-Smale systems on closed orientable 3-manifold M^3. The main goal is to present some results on interrelations between topology of M^3 and dynamics of Morse-Smale systems acting on it. We will give scetches of proofs of the statements below and discuss their applications [1].

Let
$$g = \frac{k - \ell + 2}{2},$$
where k is the number of saddles and ℓ is the number of sinks and sources of gradient-like flow (Morse-Smale diffeomorphisms) given on M^3.

Theorem 1 [2] There exists a gradient-like flow (Morse-Smale diffeomorphism) without heteroclinic trajectories (curves) on M^3 if and only if M^3 is the sphere S^3 and $k = \ell - 2$, or M^3 is the connected sum of g copies of $S^2 \times S^1$.

Theorem 2 [3] If M^3 admits gradient-like flow (diffeomorphism with tame frame of one-dimensional separatrices of saddles) then the manifold M^3 admits the Heegaard spliting of genus g.

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References

1. V. Grines , T. Medvedev, O. Pochinka, E. Zhuzhoma. On heteroclinic separators of magnetic fields in electrically conducting fluids, Physica D: Nonlinear Phenomena. Vol. 294 (2015), 1-5.

2. C. Bonatti, V. Grines, V. Medvedev, E. Pecou. Three-manifolds admitting Morse–Smale diffeomorphisms without heteroclinic curves. Topology and its Applications. 117 (2002), 335–344.

3. V. Grines, V. Medvedev, E. Zhuzhoma. New relations for Morse-Smale systems with trivially embedded one-dimensional separatrices. Sbornik: Mathematics, 2003, 194:7, 979–1007.

On constructing simple examples of three-dimensional flows with two heteroclinic cycles

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In this work we suggest a simple method for constructing Z_3-equivariant systems of ODEs in R^3 (i.e., systems, whose trajectories are invariant under the action of Z_3 on R^3) that possess two heteroclinic cycles. We assume the action of Z_3 on R^3 to be generated by cyclic permutations of coordinate axes. For several simple examples we present the analysis of the global dynamics and possible bifurcations involving heteroclinic cycles.

Cascades of bifurcations in two-parameter ecological system

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The model Hastings - Powell's ecological system of the "prey-predator-top-predator" describes a system of differential equations with parameters:

$$\zeta\dot{x} = x(1-x) - \frac{\alpha_1 xy}{1+\beta_1 x}, \dot{y} = y(\frac{\alpha_1 xy}{1+\beta_1 x} - \delta_1) - \frac{\alpha_2 yz}{1+\beta_2 y}, \dot{z} = \varepsilon z(\frac{\alpha_2 y}{1+\beta_2 y} - \delta_2). \quad (7)$$

As a bifurcation parameters considered β_1 and δ_2 and the parameters ζ, ε, α_1, α_1, β_2, δ_1 are fixed. For a singular point $O(x^*, y^*, z^*)$ which is in the region of positive x, y, z built a partition of the plane parameters β_1 and δ_2 into the regions according to the type of the coarse singular point of the linearized system. When crossing the boundary of a saddle-focus with positive real parts of a pair of complex conjugate roots going Andronov-Hopf bifurcation of birth of a stable limit cycle, followed by a cascade of period-doubling bifurcations cycle and subharmonic cascade Sharkovskii ending cycle period of the birth of three. A further change in the parameters appear in the system cycles homoclinic bifurcation cascade leading to the formation of a strange attractor. With transforms laid computing systems and evidence shows the existence of a homoclinic orbit of a saddle-focus, the destruction of which is the principal homoclinic bifurcation cascade and defined range of parameters in which it exists. Bifurcation diagrams, graphs Lyapunov exponents of saddle graphics, fractal dimension of the strange attractor. Work is executed with the use of analytical and numerical calculations Maple.

References

1. Shilnikov L., Shilnikov A., Turaev D., Chua L. Methods of qualitative theory in nonlinear dynamics: P. 2. River Edge, NJ: World Scientific, 1998, P. 393-957.
2. Magnitskii N.A., Sidorov S.V. New methods of chaotic dynamics. M.: URSS, 2004, 320 p.
3. Gurina T.A. Qualitative methods of differential equations in the theory of control of the aircrafts. M.: MAI, 2014, 160 p.

On an algorithm of recognizing the topological conjugacy classes of Morse-Smale cascades

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Mixed-mode oscillations and twin canards

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A mixed-mode oscillation (MMO) is a complex waveform with a pattern of alternating small- and large-amplitude oscillations. MMOs have been observed experimentally in many physical and biological applications, most notably in chemical reactions. We are interested in MMOs that appear in an autocatalytic system with one fast and two slow variables. The mathematical analysis of MMOs is very geometric in nature and based on singular limits of the time-scale ratios. Near the singular limit one finds so-called slow manifolds that guide the dynamics on the slow time scale. In systems with one fast and two slow variables, slow manifolds are surfaces that can be either attracting or repelling. Transversal intersections between attracting and repelling slow manifolds are called canard orbits, and they organise the observed patterns of MMOs. Our aim is to study a parameter regime where the time-scale ratio is relatively large. Here, the structure of MMO patterns is richer and different from that predicted by the theory for small time-scale ratios. To study the underlying geometry, we use advanced numerical methods based on the continuation of orbit segments defined by a suitable boundary value setup. We find that canard orbits appear in pairs, which we call twin canards. Twin canards bound regions of different numbers of small-amplitude oscillations. We also perform parameter continuations of canard orbits to identify parameter regimes of different small-amplitude oscillations.

From wild Lorenz-like to wild Rovella-like dynamics

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We consider a two-dimensional noninvertible map that was introduced by Bamón, Kiwi and Rivera in 2006 as a model of wild Lorenz-like chaos. The map acts on the plane by opening up the critical point to a disk and wrapping the plane twice around it; points inside the disk have no preimage. The bounding critical circle and its images, together with the critical point and its preimages, form the critical set. This set interacts with a saddle fixed point and its stable and unstable sets. Advanced numerical techniques enable us to study how the stable and unstable sets change as a parameter is varied along a path towards the wild chaotic regime. We find four types of bifurcations: the stable and unstable sets interact with each other in homoclinic tangencies (which also occur in invertible maps), and they interact with the critical set in three types of tangency bifurcations specific to this type of noninvertible map; all tangency bifurcations cause changes to the topology of these global invariant sets. Overall, a consistent sequence of all four bifurcations emerges, which we present as a first attempt towards explaining the geometric nature of wild chaos. Using two-parameter bifurcation diagrams, we show that essentially the same sequences of bifurcations occur along different paths towards the wild chaotic regime, and we use this information to obtain an indication of the size of the parameter region where wild Lorenz-like chaos is conjectured to exist. We further continue these bifurcations into a regime where the dynamics change from Lorenz-like to Rovella-like, that is, where the equilibrium contained in the attractor becomes contracting. We find numerical evidence for the existence of wild Rovella-like attractors, wild Rovella-like saddles and regions of multistability, where a Rovella-like attractor coexists with two fixed-point attractors.

On synchronization of hyperbolic chaotic generators and based on it communication schemes

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In the last decades the phenomenon of chaotic synchronization attracts much attention, especially from the point of view of technical applications. A special type of chaotic behavior is a hyperbolic chaos, possessing the property of robustness. A number of physically realizable models of generators with such chaotic behavior have been developed just recently [S.P. Kuznetsov. Hyperbolic chaos: A physicist's view. Springer. 2011]. The subject of this work is investigation of some of the mechanisms of transition to synchronous behavior and characteristics of dynamical and statistical properties of these mode for unidirectionally coupled hyperbolic chaotic generators, namely, one example of such a generator, which is associated to the Smale-Williams attractor. Special attention is paid to the description of robustness and hyperbolicity of the dynamics of such "master-slave"system on the road to synchrony, including the case of nonidentical by parameters subsystems. Results of numerical simulation as well as radio-electronic experiment are provided.

Moreover, in this work several schemes of chaotic communication, which functional elements (transmitter and receiver) are the generators of robust hyperbolic chaos, are proposed. These generators are associated to the dynamics of: 1) expanding circle map (Bernoulli map); 2) conservative "Arnold cat"map; 3) hyperchaotic map. To implement the confidentiality of communication the approach of nonlinear mixing of information signal to the chaotic signal of transmitter is applied. Detection of the desired information is produced in receiver by its synchronization with the transmitter. Using of the generators of robust chaos insensitive to small disturbances and perturbations is appeared to be a good choice for communication schemes.

The work was supported by RFBR (projects 14-02-00085 and 14-02-31067).

Dynamical Self-Assembly Processes

Kadina E. U., Tay M.L.

A class of dynamical models describing general features of chemical, biological and social systems is treated. In the frame of kinetic approach, such a system is characterized with concentrations of components. The concentrations satisfy a multidimensional systems of first order non-linear ordinary-derivative equations. In a general case, the equations can be analyzed using multidimensional integrated varieties and, accordngly, multidimensional attractors. Sufficient conditions for the equuilibrium uniqueness and stability are found.

If an interaction between subsystems has a hypercycle, relevant concentrations auto-oscillate. Under certain simple restrictions on parameters, the systems of equations may be integrable in quadratures.

Modeling of the employed population number nonlinear dynamics: the agent-based approach

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By now, oscillatory and complex modes of dynamics for social and economic systems are still insufficiently studied, and their substantial interpretations are debatable. Some new phenomenological principles in social-economic time series have come to light, and one of them represents structural fluctuations in the number of employed population belonging to different age groups. Modeling of these fluctuations shows that arising dynamic modes of the employed population number depend on such factors as interaction (symbiosis) or competition between the people belonging to different age groups (Khavinson, Kulakov, 2014).

At this, it still remains unclear the role of individual or group strategies in the observed fluctuations of employment. According to the neoclassical economic theory, an economic agent (a person placed in a job), should operate on a labour market in such a way that his activity would maximize his profit. Later, it was added some assumptions to this economic theory that limited rationality of the agent allowing him satisfy his social needs (Simon, 1957; Chernavskii et al, 2011; Chena, Lib, 2012; Khavinson, 2015). It is convenient to use the agent-based approach, realized in the form of a simple model, in order to investigate various employment strategies influencing the number dynamics of employed people (Kolobov, Frisman, 2013; Kolobov, 2014).

In the authorsT' model of dynamics it is considered six age groups of people (agents), employed in three conditional branches of economy. Every branch is estimated on a three-point scale, according to the salary rate, prestige and working conditions. Estimations imply that each branch would lead only at one indicator. To choose the branch, it has been considered 6 strategies, subdivided into pure and combined. The pure strategies mean that an employed person strives to maximize only one of the indicators: salary, prestige or working conditions. The combined strategies reveal a desire of the agent to choose the branch which at most satisfies two criteria: salary and prestige, salary and working conditions, or prestige and working conditions. It is also supposed that every age group is characterized by one and the same strategy of behaviour. Moving of workers between the branches depends on the choice and realization of a concrete strategy. Entry conditions in the model correspond to a uniform distribution of employed people of different age over the branches. The conducted numerical experiments show that a combination of various strategies can lead to a non-uniform distribution of workers, considering their age and conditional branches. The received results underline the importance of studying social behaviour of agents, as this factor has an influence on general dynamics and the employed population distribution.

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References

Kolobov A.N. Modeling spatial-temporal dynamics of tree stands: individual-based approach // Lesovedenie. 2014. no. 5. pp. 72-82. (in Russian).

Kolobov A.N., Frisman E.Y. Simulation of the competition for light in forest stands of the same age // Biology Bulletin. 2013. Vol. 40. no. 4. pp. 394-403.

Khavinson M.Yu. Econophysics: From Finance Analysis to the Fate of Mankind // Spatial Economics. 2015. no. 1. P. 144-166. (in Russian).

Khavinson M.Yu., Kulakov M.P. Mathematical modelling of the population dynamics of different age-groupworkers in the regional economy // Computer Research and Modeling. 2014. V.6. No. 3. P. 441-454. (in Russian)

Chernavskii D.S., Starkov N.I, Malkov S.Yu., Kosse Yu.V. and Shcherbakov A.V. On econophysics and its place in modern theoretical economics // Phys.-Usp. 2011.54.742

Chena S.H., Lib S.P. Econophysics: Bridges Over a Turbulent Current // International Review of Financial Analysis. 2012. Vol. 23. pp. 1-10.

Simon H.A. Models of Man: Social and Rational. New York: John Wiley and Sons, Inc., 1957. 279 p.

Ionization-induced wavemixing of intense laser pulses

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The present work considers the ionization-induced wavemixing by studying the excitation of radiating low-frequency (terahertz) free-electron currents by two-color laser pulses in a gas. For the two-color pulses with frequency ratio of 2, these excitation phenomena and the resulting terahertz emission were previously studied theoretically and experimentally in the context of the well-known two-color scheme of laser-plasma terahertz generation [1-4]. Here we consider the arbitrary frequency ratio in the two-color pulse.

Using the *ab initio* quantum-mechanical simulations based on the solution of the 3D time-dependent Schrödinger equation, we find the dependence of the residual current density [4,5] to which the generated terahertz field is proportional on the frequency ratio. This dependence presents multiple resonant-like peaks with maxima at rational frequency ratios $a : b$ where a and b are natural and $a + b$ is odd. Such ratios commonly occur from synchronism conditions when the high-order wavemixing (rectification) in a centrosymmetric medium is considered. The dependences of peak strengths on the intensity of an one-color component of the pump pulse (when this intensity is not too big) are also similar to those typical for high-order wavemixing. To reveal the origin of these dependences, we use the semiclassical approach [4] and obtain analytical formulas which agree well with the quantum-mechanical calculations in the parameters regions corresponding to the tunnel ionization.

Our formulas indeed demonstrate that the excitation of low-frequency currents can be treated as ionization-induced multiwave mixing, and the number of mixed waves is determined by the steepness of the ionization probability as a function of ionizing electric field and therefore depends on the intensity of pump pulses. The latter distinguishes the phenomena considered from the common high-order wavemixing. Our quantum-mechanical and semiclassical calculations show that the terahertz generation efficiency can be of the same order for the common two-color pump laser pulses (with the frequency ratio 1 : 2) and for that ones having uncommon ratios such as 2 : 3. The idea of ionization-induced wavemixing can be useful not only for design and improvement of laser-plasma schemes of terahertz generation, but also for analyses of other ultrafast strong-field phenomena based on generation of harmonics and combination frequencies.

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[1] T. I. Oh et al., Applied Physics Letters **105**, 041103 (2014).
[2] M. D. Thomson, V. Blank, H. G. Roskos, Optics Express **18**, 23173 (2010).
[3] M. Clerici et al., Physical Review Letters **110**, 253901 (2013).
[4] N. V. Vvedenskii et al., Physical Review Letters **112**, 055004 (2014).
[5] A. A. Silaev, N. V. Vvedenskii, Physical Review Letters **102**, 115005 (2009).

Towards homoclinic bifurcations and complex dynamics in the system with a “figure-eight” of a saddle

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The problem of the effect of small time-periodic forces in an asymmetric Duffing–Van-der-Pol equation close to an integrable equation with a homoclinic “figure-eight” of a conservative saddle is considered.

Applying the Melnikov analytical method and numerical simulations, the existence regions of the rough homoclinic curve of the saddle periodic motion in the control parameters plane are established. The presence of such a curve specifies complicated behavior of solutions, in other words, it leads to chaos. The region in the parameters plane which has non-smooth boundaries is detected. Homoclinic bifurcations inside this region are studied.

The problem of the structure of boundaries which separate the attraction basins of stable fixed and periodic points of the Poincaré map is also discussed. We use an algorithm for calculating the fractal dimension of such boundaries. The presence of boundaries having fractal properties results in sensitive dependence of solutions on initial conditions close to these boundaries. It makes difficult predicting the final state of the system. It is established that the fractal dimension of attraction basins boundaries of stable regimes becomes more than topological one before the first homoclinic tangency of the invariant manifolds of the saddle periodic orbit.

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Hyperbolic chaos in model systems with ring geometry

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Ring systems of different nature with uniformly hyperbolic attractors are proposed. The operation of these systems is based on phase manipulation. Finite- and infinite-dimensional models are discussed.

The first ones are ring circuits of linear or self-sustained oscillators and nonlinear elements [1,2,3]. They are arranged in such a way that the phase of the excitations undergoes an expanding circle map transformation on each full revolution through the ring. This corresponds to the appearance of the Smale-Williams attractor in the phase space of the respective Poincaré map.

Examples of distributed systems describe active medias with Smale-Williams solenoids in Poincaré cross-section [4,5,6]. They are governed by partial differential equations with periodic boundary conditions which are the modifications of the Swift–Hohenberg equation and Brusselator model, as well as to the problem of parametric excitation of standing waves by the modulated pump. Proposed models demonstrate arise and decay of spatial wave patterns on a characteristic time period. Spatial Fourier harmonics of patterns interact with each other in such a way that phases of harmonics undergo an expanding circle map. This corresponds to the appearance of the Smale-Williams attractor in infinite-dimensional phase space. Reduced finite-dimensional models for distributed systems were obtained that describe only the most important modes. Their dynamics fits well with original systems.

Proposed systems were simulated numerically. Portraits of attractors in Poincaré cross-section were obtained. Attractors of these systems are clearly of Smale-Williams type. Iteration diagrams for phases topologically correspond to expanding circle map. Lyapunov exponents were evaluated by means of the Benettin algorithm. For each model the largest exponent is positive and close to Lyapunov exponent of the expanding circle map.

For attractors of the electronic circuit models and attractors of reduced models of distributed systems, a numerical test for hyperbolicity was conducted. Distributions of the angles between the stable and unstable subspaces on the attractors have been obtained. While zero angles are inherent to non-hyperbolic attractors, histograms demonstrate absence of them and confirm that attractors of proposed models are hyperbolic.

The considered systems may be implemented in electronics or optics. Attractiveness of systems with uniformly hyperbolic attractors in a frame of possible practical application of chaos is determined by their structural stability, or robustness: the generated chaos is insensitive to variations of parameters, imperfection of fabrication, technical fluctuations in the system, etc. Such systems can find applications in information technologies and cryptography since each trajectory on uniformly hyperbolic attractor corresponds to unique infinite sequence of symbols of finite alphabet.

[1] Kruglov V.P. *Izvestiya VUZ. Applied Nonlinear Dynamics* **18** (5) (2010) 138-150 (Russian).

[2] Kruglov V.P., Kuznetsov S.P. *Commun. Nonlinear Sci. Numer. Simulat.* **16** (2011) 3219-3223.

[3] Kruglov V.P. *Izvestiya VUZ. Applied Nonlinear Dynamics* **20** (1) (2012) 124-128 (Russian).

[4] Kruglov V.P., Kuznetsov S.P., Pikovsky A. *Regular and Chaotic Dynamics* **19** (4) (2014) 483-494.

[5] Kruglov V.P. V.P., Kuznetsov A.S., Kuznetsov S.P. *Rus. J. Nonlin. Dyn.* **10** (3) (2014) 265-277 (Russian).

[6] Kruglov V.P. *Izv. VUZ. Appl. Nonlinear Dynamics* **22** (6) (2014) 79-93 (Russian).

The dynamics of a system of phase oscillators depending on coupling strength and other parameters

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The current study aims to examine the dynamics of two coupled phase oscillators. We consider a general case of non-identical elements with their individual characteristics represented by parameters of the system. This work is motivated by multiple problems of neuroscience where different types of neurons influence each other's behaviour by means of various interconnections. Thus, it appears to be vital to examine how the coupling strength can influence the systems' dynamics. Not only can organisation of the connection between the elements change the systems' behaviour, but in many cases it is considered to be more influential than individual parameters of the elements as well. Thus, the main object is to describe regimes of the elements' activity depending on their characteristics and coupling strength.

Nonlinear dynamics of the number of different age specialists in the regional economy

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The questions about the stability of socio-economic systems are very topical in modern economics. One such system is the regional labor market. A variety of socio-economic interactions between employed population can provoke a strong polarization of the role of age groups on the labor market, which is likely to have a negative impact on quality of life, economic development and attractiveness of the region.

In modern science, the interaction of various objects is studied at a high level with the use of nonlinear mathematical models (Zhang, 1996; Weidlich, 2000; Milovanov, 2001). Using shaped approaches to the study of complex interaction of objects, we propose to examine the interaction of different age professionals specialists in the regional economy in the key of nonlinear dynamics.

It was found that the complex three-dimensional structure in the phase space is formed through a cascade of period doubling cycle or by a gradual noise on the limit cycle, which demonstrates the system's sensitivity to small deviations of the initial conditions and the fundamental impossibility of medium-and long-term forecasting.

Unpredictable changes in the number of different age specialists of can be explained by unfavorable situation on the labor market: the migration flow prevails over the outflow of only one cohort, all age groups are characterized by a significant influx of economically inactive. It should be noted that complex periodic modes appear in a fairly narrow areas of the parameter space and have varying degrees of stability to change specific parameters. In the applied aspect, it means that to eliminate or smoothing fluctuations in the labor market can be regulated by the bifurcation parameter of the system.

The model equations used to describe the dynamics of the interaction of the employed population in the Jewish Autonomous Region (JAR) of the three age groups: 16-29, 30-49 and 50 and over, according to 1997-2010.

The proposed model reveals the nonlinear effects in the development of the labor market. It's necessary to considered in the management of the region and the development of forecasts.

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References

Milovanov V.P. Nonequilibrium socio-economic systems: synergy and self-organization. Moscow: Editorial URSS, 2001. 264 p. (in Russian).

Weidlich W. Sociodynamics: A Systematic Approach to Mathematical Modelling in the Social Sciences. Harwood Academic Publishers. 2000. 392 p.

Zhang W.-B. Synergetic economics: time and change in nonlinear economics. Springer-Verlag, 1991, p. 246.

Clustering in the coupled Ricker population model

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In various sciences there is a problem of studying the features coupled oscillating elements dynamics. For example these are systems of coupled neurons, lasers, alternating current generators or migration-related populations. One kind of this model is a system of coupled logistic maps or coupled map lattice (CML). The single map of CML demonstrates the transition to chaos through period doubling.

To describe the dynamics of biological population or metapopulation the each patch is modeled a logistic map (Ricker model or Ricker map) and relationship between neighboring populations (migration flows) is a dissipative coupling between neighboring oscillators on plane lattice. As result the coupling between them apparently can not be global, leading to the complex dynamics mode. For instance besides the phenomena of multistability and synchronization there is a "boundary effect" which makes it impossible to complete synchronization, especially for large values of the migration coefficient (coupling force). As consequence the initially identical and symmetrically coupled maps in different clusters may fluctuate with the different amplitudes, phases or periods.

In this abstract the phenomenon of clustering and multistability in coupled Ricker population model are studied.

It is shown the forming of clusters is complex depends on an initial distribution of individuals in flat area. It is described the bifurcation scenario of two non equal clusters formation correlated clusters with ordered phase. As result it is shown its attraction basins of these clutters coincide partially with basins of antiphase modes in system of two non-symmetrical and not identical coupled Riker maps. It is found the phase space consists of a large number of embedded in each other basins of similar clustering phase. These modes are kind of transition states between coherent mode and phase with two equal clusters. In addition to these modes's domains of attraction the basins of three-cluster states are found. Basins of these modes are union of attraction domains of two-cluster phases with a different number of occupation numbers.

Moreover the dynamic of two coupled non-identical Ricker maps on the average approximates the dynamic of two non equal clusters. It is shown the differences between each of these maps depend on clusters size and number of direct connections between clusters. The study of non-identical maps Ricker dynamic modes makes it possible to suggest what dynamic modes are possible for two unequal clusters. As a result it is found the ordered phase with a fixed size and location of clusters has a multistable nature. It means it can be implemented in several ways. These ways are not only the order of the system states, i.e. the phase of oscillations, but a fundamentally different type of dynamics: various cycle lengths and location attractor in the phase space. Furthermore the formation of two or more clusters has the same mechanism as bifurcation of the generation asynchronous modes in the approximating system. But further bifurcations do not always coincide.

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Parametric generators of chaos

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Several schemes of parametric generators of chaotic oscillations are suggested and analyzed. Among them there are systems with Lorenz-type attractor and attractors represented by a kind of Smale-Williams solenoid in Poincaré map [1-3]. Beside the mathematical models and numerical computations, concrete electronic schemes are considered, and results of their simulation using software product Multisim are presented and discussed. The work is supported in part by RFBR grants No 14-02-00085 and 15-02-02893 and by the grant for leading scientific schools NSh-1726.2014.2.

1. Kuznetsov S.P. Parametric oscillator of robust chaos: Circuit implementation and simulation using the program product Multisim // Vestnik SGTU. 2014. i 3 (76). P. 34-46. (In Russian.)
2. Kuznetsov S.P., Kuznetsov A.S., Kruglov V.P., Hyperbolic chaos in systems with parametrically excited patterns of standing waves // Rus. J. Nonlin. Dyn. 2014. Vol. 10. No. 3. P. 265-277. (In Russian.)
3. Kuznetsov S.P. Generator of chaos on varactor diode with decay saturation of the parametric instability // Technical Physics (submitted).

Dynamics and bifurcations in a simple quasispecies model of tumorigenesis

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Cancer is a complex disease and thus is complicated to model. However, simple models that describe the main processes involved in tumoral dynamics, e.g., competition and mutation, can give us clues about cancer behaviour, at least qualitatively, also allowing us to make predictions. Here, we analyse a simplified quasispecies mathematical model given by differential equations describing the time behaviour of tumor cell populations with different levels of genomic instability. We find the equilibrium points, also characterizing their stability and bifurcations focusing on replication and mutation rates. We identify a transcritical bifurcation at increasing mutation rates of the tumor cells. Such a bifurcation involves a scenario with dominance of healthy cells and impairment of tumour populations. Finally, we characterize the transient times for this scenario, showing that a slight increase beyond the critical mutation rate may be enough to have a fast response towards the desired state (i.e., low tumour populations) by applying directed mutagenic therapies.

This is a joint work with Josep Sardanyés[†] and Vanessa Castillo[‡].

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On topological classification of diffeomorphisms of 3-manifold with one-dimensional surface basic sets

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In this report we consider a class of diffeomorphisms of 3-manifold, such that each diffeomorphism fron this class is a locally direct product of a DA-diffeomorphism of 2-torus and rough diffeomorphism of the circle. We find algebraic criteria for topological conjugacy of the systems. It is proved that the class of topological conjugacy of such diffeomorphism is completely determined by combinatorial invariants, namely hyperbolic automorphism of the torus, a subset of its periodic orbits, the number of periodic orbits and the serial number of the diffeomorphism of the circle.

The study was partially supported by the Russian Foundation for Basic Research (grants 13-01-12452-ofi-m, 15-01-03687-a), RNF grant 14-41-00044. The author also thanks the Ministry of Education and Science of the Russian Federation (Project 1410).

Finiteness and existence of attractors and repellers on sectional hyperbolic sets

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Actions of symplecting groups of diffeomorphisms of orientable surfaces on spaces of smooth functions

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Let M be a closed orientable surface and $D(M)$ be the group of diffeomorphisms of M acting from the right on $C^\infty(M)$ by the rule: the result of the action of $h \in D(M)$ on $f \in C^\infty(M)$ is the composition $f \circ h : M \to R^1$. Let $S(f) = \{f \circ h = f \mid h \in D(M)\}$ and $O(f) = \{f \circ h \mid h \in D(M)\}$ be respectively the stabilizer and the orbirt of $f \in C^\infty(M)$. Denote by $S_{\mathrm{id}}(f)$ the path component of id_M in $S(f)$ and by $O_f(f)$ the path component of f in $O(f)$. In a recent series of papers the author described the homotopy types of $S_{\mathrm{id}}(f)$ and $O_f(f)$ for a large class of smooth functions on M which includes all Morse functions.

Let ω be any symplectic 2-form on M. Then one can consider the restriction of the above action of $D(M)$ on the groups $Symp(M,\omega)$ of symplectic diffeomorphisms of M. For $f \in C^\infty(M)$ let $S(f,\omega)$ and $O(f,\omega)$ be the corresponding stabilizer and orbit of f with respect to $Symp(M,\omega)$. Then $S(f,\omega) = S(f) \cap D(M)$ and $O(f,\omega) \subset O(f)$. Similarly, let $S_{\mathrm{id}}(f,\omega)$ the path component of id_M in $S(f,\omega)$ and by $O_f(f,\omega)$ the path component of f in $O(f,\omega)$.

Theorem *Let $f : M \to R^1$ be a C^∞ Morse function. Then there exists a symplectic 2-form ω on M such that both inclusions $S_{\mathrm{id}}(f,\omega) \subset S_{\mathrm{id}}(f)$ and $O_f(f,\omega) \subset O_f(f)$ are weak homotopy equivalences.*

Renormalization and Universality for Rotation Sets in Lorenz-like Systems

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Geometric models of Lorenz-like systems as well as models in the form of countable topological markov cchains are considered. By using rotation sets of the models and of their renormalizations we study behavior of rotation sets in one-parameter families of multidimensional perturbations of one-dimensional maps of Lorenz type. More precisely, let

$$\Phi_\lambda(y_n, y_{n+1}, \ldots, y_{n+m}) = 0, \; n \in \mathbf{Z},$$

be a difference equation of order m with parameter λ. It is asuumed that the non-perturbed operator Φ_{λ_0} depends on two variables, i.e., $\Phi_{\lambda_0}(y_0, \ldots, y_m) = \psi(y_N, y_M)$, where $0 \le N, M \le m$ and ψ is a piecewise monotone piecewise C^2-function. It is also assumed that for the equation $\psi(x,y) = 0$, there is a branch $y = \varphi(x)$ which represents a one-dimensional Lorenz-type map. We prove approximation results for the problem on continuous dependence of the rotation set under multidimensional perturbations. Numerical results show universality phenomena in the bifurcation structure responsible for birth of nontrivial rotation intervals with respect to the maps and also to their renormalizations. Our technique is based on approximations of topological entropy and maximal measures represented by countable topological Markov chains [1] and also, on continuation of chaotic orbits for perturbations of singular difference equations [2].

Besides, we compare the results for behavior of nonwandering orbits of the systems under consideration with ones of generalized polynomial Hénon maps.

[1] Malkin M.I. *Countable Topological Markov Chains with Meromorphic Zeta Functions.* Random & Computational Dynamics 2 (1994), no. 3-4, 247-259.

[2] Juang J., Li M.-C., Malkin M.I. *Chaotic difference equations in two variables and their multidimensional perturbations.* Nonlinearity, 21 (2008), 1019-1040.

On classification of Morse-Smale systems with few non-wandering points

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We consider Morse-Smale systems whose non-wandering set consists of three fixed points. We formulate necessary and sufficient conditions for such systems to be conjugacy.

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On transitory systems

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We consider the two-dimensional non-autonomous systems one whose time-dependence is confined to a bounded interval. Following [1], such systems we call transitory.

We consider a subclass of transitory systems that are close to nonlinear conservative. As an example the system

$$\begin{aligned} \dot{x} &= y - \omega f(t), \\ \dot{y} &= x - x^3 + \varepsilon(p - x^2)y \end{aligned} \tag{8}$$

is investigated. Here ε is a small parameter, p, ω are parametres. Transition function $f(t)$ is the smooth function, satisfying the condition

$$f(t) = \begin{cases} 0, & t \le 0, \\ 1, & t \ge \tau. \end{cases}$$

The system (1) at $t \le 0$ takes place in the flutter problem [2,3]. In the conservative approximation of this problem ($\varepsilon = 0$) we identified a quantitative evaluation (a measure) of transport between coherent structures. In the nonconservative approximation we consider the impact of transitory shift to setting of one or another attractor. We give probabilities of changing a mode (stationary to auto-oscillation). Also we point out to the possibility of oscillation stabilization as a result of the transitory shift.

[1] Mosovsky B.A., Meiss J.D. Transport in Transitory Dynamical Systems, *SIAM J. Applied Dyanamical Systems*, 2011, vol. 10, no. 1, pp. 35-65.

[2] Holmes Ph.J. Bifurcations to divergence and flutter in flow-induced oscillations: a finite dimensional analysis, *Journal of Sound and Vibration*, 1977, vol. 53, no. 4,471-503.

[3] Guckenheimer J., Holmes Ph. *Oscillations, Dynamical Systems and Bifurcations of Vector Fields*, Springer, 1983.

Phase diffusion in unequally noisy coupled oscillators

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We consider the dynamics of two directionally coupled unequally noisy oscillators, the first oscillator being nosier than the second oscillator. Examples include spontaneous stochastic oscillations of sensory hair cells and synaptically coupled neurons. We derive analytically the phase diffusion coefficient of both oscillators in a heterogeneous setup (different frequencies, coupling coefficients and intrinsic noise intensities) and show that the phase coherence of the second oscillator depends in a non-monotonic fashion on the noise intensity of the first oscillator: as the first oscillator becomes less coherent, i.e. worse, the second one becomes more coherent, i.e. better. This surprising effect is related to the statistics of the first oscillator which provides for the second oscillator a source of noise, which is non-Gaussian, bounded, and possesses a finite bandwidth.

Features of dynamics of the implicit maps. Conservative and near conservative case

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In the theory of chaos it is well-known the special class of dynamical systems - the discrete maps. There are reversible in time maps, when each point in phase space has only one preimage, and nonreversible naps, when several preimages may be occur. One can imagine such a discrete model in which there is several solutions in forward as in backward time. These systems have an implicit evolution operator

$$g(z_{n+1}, z_n) = 0. \qquad (9)$$

The maps of such type are abstract models, as well as the explicit but nonreversible maps. In work [S.R. Bullett et al //Physica 19D, 1986, P.290] an implicit map of the complex plane with special unitary symmetry is described. The conservative trajectories representing a stochastic web are found in the phase p lane of this system. In present work, we make an attempt to study the transition from an nonreversible Mandelbrot map to an implicit one, until conservative limit.

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Attractors of skew products with circle fiber

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Partially hyperbolic skew products over Anosov diffeomorphisms and their "younger brothers", step skew products over topological Markov chains, provide interesting dynamics even for one-dimensional fiber. For the interval fiber there are strange examples in border-preserving case : intermingled basins [1] and thick (i.e. having positive but not full measure) attractor [2]. In non-border-preserving case there is a finite collection of alternating attractors and repellers, each of them is a bony graph [3]. The Milnor attractor is the smallest closed subset, containing ω-limit sets of almost all points. It is unknown whether the Milnor attractor is the union of these bony graphs (or, equivalently, whether it is asymptotically stable).

We will discuss the following result in the circle fiber case: the Milnor attractor is Lyapunov stable and not thick; either the skew product is transitive or the nonwandering set has zero measure. Main ingredients of the proof are the semicontinuity lemma and the fact that ω-limit set of a generic point is saturated by unstable leaves.

[1] I. Kan, *Open sets of diffeomorphisms having two attractors, each with everywhere dense basin,*

[2] Yu.S. Ilyashenko, *Thick attractors of boundary preserving diffeomorphisms,* Indagationes Mathematicae, 2011

[3] V. Kleptsyn, D. Volk, *Physical measures for nonlinear random walks on interval,* Mosc. Math. J., 2014

Criteria of the birth of Lorenz attractors

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Generic criteria of the birth of Lorenz attractors for systems with two homoclinic loops were formulated in [1]. Based on them, explicit conditions on the parameters of the homoclinic orbits were developed by L.P. Shilnikov in [2, 3]. In the present work these conditions were extended to the situation when, unlike the Lorenz-like systems, there is no symmetry present. In particular, the considered cases include the so-called *semi-orientable Lorenz attractors.*

References

[1] V.S. Afraimovich, V.V. Bykov, L.P. Shilnikov. On attracting structurally unstable limit sets of Lorenz attractor type. Trans. Mosc. Math. Soc., 44 (1982), 153???216.

[2] L.P. Shilnikov, The bifurcation theory and quasihyperbolic attractors, Uspekhi Mat. Nauk 36:4 (1981), 240–241.

[3] L.P. Shilnikov. Bifurcations and Strange Attractors. Preprint: http://arxiv.org/abs/math/0

On topological classification of structurally stable dynamical systems

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In this review we will focus on a traditional subject for Nizhny Novgorod school of non-linear oscillations – the topological classification of the dynamical systems. This section of the qualitative theory is dedicated to finding of properties are preserving under the topological conjugacy and which are sufficient to construct a homeomorphism sending trajectories of one system to the trajectories of other one. We restrict attention to the structurally stable systems – systems that do not change a class of the topological conjugacy after a small perturbation. First such continuous dynamical systems on the plane, as well as the notion of structural stability (roughness), were described by A. Andronov and L. Pontryagin in 1937. In the same year they were classified by Nizhny Novgorod mathematicians E. Leontovich-Andronov and A. Mayer. To date, significant progress is achieved in the topological classification of the structurally stable systems, both discrete and continuous, both regular and chaotic dynamics. In the lectures we give results in this direction, starting with the dynamic systems on a circle and finishing with systems on multidimensional manifolds.

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Two-parameter bifurcation study of the regularized long-wave equation

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We perform a two-parameter bifurcation study of the driven-damped regularized long-wave equation by varying the amplitude and phase of the driver. Increasing the amplitude of the driver brings the system to the regime of spatiotemporal chaos (STC), a chaotic state with a large number of degrees of freedom. We identify four distinct routes to STC; they depend on the phase of the driver and involve boundary and interior crises, intermittency, the Ruelle–Takens scenario, the Feigenbaum cascade, an embedded saddle-node, homoclinic and other bifurcations.

Feynmanons in gravitational waves in fluid

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In this report we consider linear gravitational waves in incompressible infinitely deep fluid on the plane z = 0 corresponding to unperturbed surface of fluid. In this case one proves to find equation of hyperbolic type for potential of velocity of fluid at z = 0. By the same way as in report [1] we reduce this equation to the vector Schrödinger type equation with two spatial variables. Green's matrix of this vector equation is expressed by Feynman integral too. This path integral with four-dimensional phase space has fractal structure of its trajectories described in detail in paper [2]. Nonlocal quantum field theory for two-dimensional potential of velocity of fluid also is presented. In accordance with our general line [1, 2] we introduce the new kind of feynmanon as object moving along fractal trajectories in considered Feynman integral. We have called this quasiparticle by "surface hydron". The results of this work can be easily extended on a number of more complicated situations for linear gravitational waves namely on the case of incompressible infinitely deep fluid with surface tension, on the case of incompressible fluid with finite deep, on the case of layers of incompressible fluids with different densities and so forth.

References:

1. Potapov A. A., Rassadin A. E. Fractals and path integrals in three-dimensional wave equation // Book of abstracts of International Conference-School "Infinite-dimensional dynamics, dissipative systems, and attractors" (Nizhny Novgorod, Russia, July 13-17, 2015) p. 26-27.

2. A. A. Potapov, A. E. Rassadin. Feynman integrals, fractal paradigm and new point of view on hydroacoustics // Eurasian Physical Technical journal (in print).

Plane waves, Lie groups and Feynman integrals

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Using the representation of plane wave in n-dimensional space by means of Feynman integral [1] we show that symmetry of this wave is closely connected with semidirect product of real symplectic group Sp(2n,R) and Heisenberg-Weyl group H(n).

If $n = 4$ then phase space of corresponding Feynman integral is eight-dimensional space. It means that in this case four momenta and four coordinates one can consider as eight components of octonion. It is well-known that five exceptional simple Lie groups $G2, F4, E6, E7$ and $E8$ are related to octonions [2]. Furthermore group $G2$ is group of symmetry for octonionic algebra. On the other side simple Lie groups $G2$ is necessary in theory of strings in order to reduce eleven dimensions of M-theory to four dimensions of our space-time [2].

If one introduce eight-dimensional lattice in the phase space then random walk on this lattice is subset of set of phase trajectories in the Feynman integral. In particular an example of such trajectory is eight-dimensional Peano curve filling eight-dimensional hypercube [3].

As in three-dimensional case [4] dynamics of momenta may obey to succession map. If this map satisfies to conditions of Williams-Hatchinson theorem [3] then momenta form four-dimensional fractal set. In succession map may take place chaotic behaviour too. On the other hand four momenta one can consider as four components of quaternion. If we split four components of this quaternion on two complex variables then our succession map for momenta can be reduced to holomorphic map possessing by very untrivial dynamics.

References:

1. Potapov A. A., Rassadin A. E. Feynman Integrals, Fractal Radio Systems, or Do We Understand What is Radio Engineering? // Proc. From 17th Int. Conf. and Exh. “Digital Signal Processing and Its Applications”, Moscow, 2015, V.1, pp. 103-109.

2. Boya L. J. Octonions and M-theory. http://arXiv.org/abs/hep-th/0301037 .

3. Potapov A. A., Gulyaev Yu. V., Nikitov S. A. and alt. The modern methods of image processing. 2008, M., Fizmatlit, 496 p.

4. Potapov A. A., Rassadin A. E. Fractals and path integrals in three-dimensional wave equation // Book of abstracts of Int. Conference-School “Infinite-dimensional dynamics, dissipative systems, and attractors” (Nizhny Novgorod, Russia, July 13-17, 2015) p. 26-27.

Multistability in quasiperiodically driven Ikeda map

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It is well known that dynamical systems with weak dissipation can demonstrate a great number of attractors [1]. In this work mechanisms of phase space structure changes for the system with weak disipation while the quasiperiodical influence to the system is induced have been investigated using the Ikeda map [2].

The investigated map is given by

$$E_{n+1} = A(1 + \varepsilon \sin(\Omega \cdot \varphi \cdot n)) + BE_n \exp(i|E_n|^2 + i\varphi),$$

where A is control parameter, B is parameter of dissipation, ε is amplitude of external influence, Ω is frequency of influence, φ is phase.

In the work the structure of coexisting attractors and their evolution while ε is changed in the case of weak dissipation are investigated. It is shown that the number of coexisting attractors decreases in comparison with the case of absence of external influence. It occurs due to the finite size of torus attractor in contrast to periodical attractor. The attractor number dependence on the ε is studied for different values of A. The evolution of attractor basins boundaries while quasiperiodical influence is appended has been studied.

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References

1. *Feudel U. et al.* //Phys. Rev. E. 1996. Vol. 54, пї- 1. P. 71.

2. *Ikeda K., Daido H., Akimoto O.* //Phys. Rev. Lett. 1980. Vol. 45. P. 709.

Dynamics near a homoclinic network with a bifocus

A.Rodrigues

Bifurcations of first integrals in the Kowalevski – Sokolov case

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The phase topology of the integrable Hamiltonian system on $e(3)$ found by V.V.Sokolov (2001) [1] and generalizing the Kowalevski case (1889) [2] is investigated. The generalization contains, along with a homogeneous potential force field, gyroscopic forces depending on the configurational variables. Relative equilibria are classified, their type is calculated and the character of stability is defined. The Smale diagrams of the case are found and the classification of iso-energy manifolds of the reduced systems with two degrees of freedom is given. The set of critical points of the complete momentum map is represented as a union of critical subsystems; each critical subsystem is a one-parameter family of almost Hamiltonian systems with one degree of freedom. For all critical points we explicitly calculate the characteristic values defining their type. We obtain the equations of the surfaces bearing the bifurcation diagram of the momentum map. We give examples of the existing iso-energy diagrams with a complete description of the corresponding rough topology (of the regular Liouville tori and their bifurcations).

The work is partially supported by the RFBR (grant No. 15-41-02049).

References

[1] Sokolov V.V. A new integrable case for the Kirchhoff equations, *Theoretical and Mathematical Physics*, 2001, vol. 129, no. 1, pp. 1335–1340.

[2] Kowalevski S. Sur le probléme de la rotation d´un corps solide autour d´un point fixe, *Acta Mathematica*, 1889, vol. 12, no. 1, p. 177–232.

Strange nonchaotic attractors in quasiperiodically driven Ikeda map with weak dissipation

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It is known that the quasiperiodically driven systems demonstrate some specific features of dynamics, i.e. the realisation of strange nonchaotic attractors (SNA). The dynamics of quasiperiodically driven dissipative systems is well studied now [1]. It seems interesting to consider the peculiarities of these phenomena in the systems with weak dissipation, in particular to reveal the SNAs and corresponding regions in the parameter plane. Recently [2] it was shown that SNA exists in the map with weak dissipation but no results consider the size of corresponding region on the parameter plane and its dependence on dissipation parameter is known.

In this work we consider the well-known Ikeda map [3] driven by signal with incommensurate frequency:

$$z_{n+1} = A(1 + \varepsilon \sin 2\pi\theta_n) + Bz_n e^{i|z_n|^2}, \theta_{n+1} = \theta_n + w.$$

The frequency ratio w was taken equal to golden ratio $w = (\sqrt{(}5) - 1)/2)$ and the structure of the "nonlinearity parameter A - the driving amplitude ε" plane was studied for different dissipation values B by calculation the Lyapunov exponents.

The identification of SNAs was based on the values of largest Lyapunov exponent and visual analysis of its structure both with the rational approximants [1] method. It was revealed that the band of parameter A values in which the SNA exists decreases significantly with the decrease of dissipation.

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[1] U.Feudel, S. Kuznetsov, A.Pikovsky. Strange Nonchaotic Attractors. World Scientific, 2006

[2] S. Bilal, R. Ramaswamy //Phys. Rev.E, v.87, 034901 (2013)

[3] Ikeda K., Daido H., Akimoto O. //Phys. Rev. Lett., 1980, v.45, p.709.

Experimental study of transition to the robust chaos in the Kuznetsov generator

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An example of a dynamical system of ODEs demonstrating the regime of robust hyperbolic chaos was first introduced in the work of Kuznetsov [1], and later the radiotechnical device demonstrating such type of dynamics was constructed [2]. Such generator of chaos could be very useful for applications due to the robustness of its dynamics.

In the present work we introduce another example of the radiotechnical Kuznetsov generator and investigate its parameter plane experimentally at frequencies of 50 kHz.

In presented system Smale-Williams-like solenoidal attractor exists in the wide domain on the parameter plane. We calculate different characteristics for the trajectories in this domain, particularly, the iteration diagram for the phase of the oscillations, which is similar to the Bernoulli map, power spectra, local Lyapunov exponents. It is possible to obtain two different scenarios of the SW-like attractor formation: via the quasiperiodicity destruction and via the transformation of the Feigenbaum chaotic attractor.

The work was supported by Russian Foundation for Basic Researches (projects 14-02-00085 and 14-02-31067).

References

1. *Kuznetsov S. P.* // Phys. Rev. Lett. 2005. Vol. 95. P. 144101.
2. *Kuznetsov S. P., Seleznev E. P.* // J. Exper. Theor. Phys. 2006. Vol. 102, No 2. P. 355.

Dynamics of ensemble of active Brownian particles interacting via Morse potential forces

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Dynamics of an one-dimensional ring of active Brownian particles interacting via morse potential forces is under study. Nonlinear friction defined in sense of Rayleigh. Density of ensemble can be varied.

In case of low density the statistics of clusters formation is studied. The evolution of border of bifurcational transition between ordered and disordered states is considered in a parameter space, particularly with different quantity of particles.

In high density ensemble the inception of solitons and their steady states are studied.

In both cases the transformation of particles velocity distribution is analyzed in terms of ensemble density and other parameters.

Evidence of a strange nonchaotic attractor in the El Nino dynamics

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Basing on a mathematical idea about the so-called strange nonchaotic attractor in the quasi-periodically forced dynamical systems, re-analyses data are considered. It is found that the El Nino - Southern Oscillation (ENSO) is driven not only by the Sun-induced periodic heating (seasonal and $\sim$ 11.2-year sunspot cycle), but also by two more external periodicities (incommensurate to the annual period) associated with the $\sim$ 18.6-year lunar-solar nutation of the Earth rotation axis, and the 14-month Chandler wobble in the Earth's pole motion. Because of the incommensurability of their periods all three forces affect the system in inappropriate time moments. As a result, the ENSO time series look to be very complex (strange in mathematical terms). The power spectra of these series reveal numerous peaks located at the periods that are multiples of the above periodicities as well as at their sub- and super-harmonic. In spite of this strangeness, a mutual order seems to be inherent to these time series and their spectra. This order reveals itself in the existence of a scaling of the power spectrum peaks and respective rhythms in the ENSO dynamics that look like the power spectrum and dynamics of the strange nonchaotic attractor. It means there are no limits to forecast ENSO, in principle. In practice, it opens a possibility to forecast ENSO for several years ahead.

Fermi acceleration in billards with holes

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Fermi acceleration is the study of energy growth of particles moving within a closed billiard undergoing elastic collisions with its walls. This model can be used to study plasma heating in tokamaks, mesoscopic systems, etc. However, many of these physical systems are not closed but have an exchange of particles across the boundaries. For example, in modeling of RF heating in tokamaks, Fermi acceleration is applicable to electrons in the sheath region but these electrons keep moving back and forth between the sheath and bulk. Our study of Fermi acceleration in billiards with a small hole is the first step towards a better understanding of such open systems. We assume that the particles do not collide with each other and the hole size is small enough so that the particles remain within the billiard for a sufficiently long time. Our primary result is a theoretical estimate of the net energy flow through the hole and we have verified this through numerical simulations [arxiv:1504.00897]. We find that the heat production strongly depends on the type of the Fermi accelerator. A leaky ergodic accelerator (one which has a single ergodic component, and has quadratic-in-time energy growth without the hole) produces a much lower energy flow than a leaky multi-component accelerator (which has exponential-in-time energy growth without the hole). Specifically, the energy gain is independent of the hole size if the accelerator is ergodic, whereas the energy flow may be significantly increased by shrinking the hole size in the exponential accelerator.

Work done jointly with : Gelfreich V, Rom-Kedar V and Turaev D.

Dynamical chimeras in a ring of oscillators with local coupling

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Chimera states in oscillatory ensembles are of great interest for the last time [1,2]. The typical feature of chimera regimes is nonlocal character of coupling of elements in ensemble. Beside of chimeras in ensembles the so named virtual chimeras can exist in the single systems with time delayed feedback [3]. It is known about the analogy between the system with delayed feedback and the spatially distributed system with periodic boundary conditions. Using this analogy one can obtain the dynamical chimera in the ring of oscillators with the local connections between the elements. Its properties should be the same as that of the chimera in the system with delayed feedback. In this work we construct the chimera similar to the described in [3] arising in the ring of oscillators with the local unidirectional nonlinear coupling. Beside of this we studied the chimera-like regimes in a ring of classical Du?ng?s oscillators with the simple local unidirectional coupling. The evolution of the traveling waves and the chimera-liked structures are investigated with the variation of the coupling strength.

[1] Y. Kuramoto, D. Battogtokh, Nonlinear Phenom. Complex Syst. 5, 380 (2002).
[2] M.J. Panaggio, D.M. Abrams,. Nonlinearity 28 (3), R67 (2015).
[3] L. Larger, B. Penkovsky, and Y. Maistrenko, Phys. Rev. Lett. 111, 054103 (2013).

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Lyapunov unstable Milnor attractors

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The *Milnor attractor* of a dynamical system is the smallest closed set that attracts almost every point. This notion of a global attractor is well defined for homeomorphisms of a compact metric measure space but the talk will be restricted to diffeomorphisms of smooth Riemannian manifolds.

Recall the classical notion of *Lyapunov stability*: a subset of the phase space is stable if for any its neighbourhood there is a smaller neighbourhood such that the orbits which start at the latter never quit the first one. It is easy to give an example of a diffeomorphism whose Milnor attractor is Lyapunov unstable and therefore it is only natural to ask how typical this instability of attractors can be.

It turns out that it is a locally topologically generic phenomenon, i.e., there are open domains in the space of diffeomorphisms such that residual (Baire 2^{nd} category) subsets of these domains consist of diffeomorphisms with Lyapunov unstable Milnor attractors. The crucial ingredient here is the existence of so-called *Newhouse domains*, open subsets of the space of diffeomorphisms where the maps exhibiting a homoclinic tangency associated with a continuation of a single hyperbolic saddle are dense and where coexistence of infinitely many sinks is generic. Whenever there is a Newhouse domain where the tangencies are associated with a *sectionally dissipative* saddle, a topologically generic diffeomorphism in this domain has an unstable Milnor atractor.

Another approach yields the following global statement. For a topologically generic C^1-diffeomorphism of a closed manifold, either any *homoclinic class* admits a *dominated splitting* (which is a very mild notion of hyperbolic-like behaviour) or the Milnor attractor is unstable for this diffeomorphism or for its inverse.

Global stabilisation for damped–driven conservation laws

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We consider a multidimensional conservation law with a damping term and a localised control. Our main result proves that any (non-stationary) solution $u(t, x)$ can be exponentially stabilised in the following sense: for any initial state one can find a control such that the difference between the corresponding solution and the function $u(t)$ goes to zero exponentially fast in an appropriate norm. As a consequence, we prove global exact controllability to solutions of the problem in question. We also establish global approximate controllability to solutions with the help of low-dimensional localised controls. This is joint work with S. Rodrigues.

Chimera States in Ensemble of Non-Locally Coupled Anishchenko – Astakhov Self-Sustained Oscillators

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The ring of non-locally coupled Anishchenko – Astakhov self-sustained oscillators is under study. The equations which are described the ring are following:

$$\begin{cases} \dfrac{dx_i}{dt} = mx_i + y_i - x_i z_i + \dfrac{\sigma}{2P}\sum\limits_{j=i-P}^{i+P}(x_j - x_i), \\ \dfrac{dy_i}{dt} = -x_i + \dfrac{\sigma}{2P}\sum\limits_{j=i-P}^{i+P}(y_j - y_i), \\ \dfrac{dz_i}{dt} = g(\Phi(x_i) - z_i), \end{cases} \\ \Phi(x) = \frac{x}{2}(x + |x|), \\ i = 1,\ 2,\ \ldots,\ N, \tag{10}$$

where m and g — control parameters, σ — coupling strength, P — number of coupled neighbors, N — number of elements in a ring.

In a regime of weak chaos we demonstrate the existence of chimera states, traveling waves and various synchronization regimes in dependence on chosen initial conditions and parameters values. The reasons of occurrence of one or another regime are under discussion.

The study was partially supported by RFBR (research project No. 14-52-12002) and by the Russian Ministry of Education and Science (project code 1008).

Falling Motion of a Circular Cylinder Interacting Dynamically with Point Vortices

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The problem of falling motion of a body in fluid has a long history and was considered in a series of the classical and modern papers. Some of the effects described in the papers, such as periodic rotation (tumbling), can be encountered only in viscous fluids and thus demand for their proper treatment the use of the Navier - Stokes equations with boundary conditions specified on the body's surface. As a rule, such problems are hardly amenable to analytical analysis and can be addressed only numerically.

Another approach is to use (instead of the exact Navier - Stokes equations) some phenomenological ODE models which capture the viscous effects qualitatively.

In this paper we study the influence of the vorticity on the falling body in a trivial setting: a body (circular cylinder) subject to gravity is interacting dynamically with N point vortices. The circulation around the cylinder is not necessarily zero. So the model we consider here is exact and, at the same time, not so despairingly complex as most of the existing models are. The dynamical behavior of a heavy circular cylinder and N point vortices in an unbounded volume of ideal liquid is considered. The liquid is assumed to be irrotational and at rest at infinity. The circulation about the cylinder is different from zero. The governing equations are presented in Hamiltonian form. Integrals of motion are found. Allowable types of trajectories are discussed in the case of single vortex. The stability of finding equilibrium solutions is investigated and some remarkable types of partial solutions of the system are presented. Poincare sections of the system demonstrate chaotic behavior of dynamics, which indicates a non-integrability of the system.

In a case of zero circulation using autonomous integral we can also reduce the order of the system by one degree of freedom. Unlike nonzero circulation and the absence of vortices when the cylinder moves inside a certain horizontal stripe it is shown that in a presence of vortices and with circulation equal to zero vertical coordinate of the cylinder is unbounded decreasing. We then focus on the numerical study of dynamics of our system. In a case of zero circulation trajectories are noncompact. The different kinds of the scattering function of the vortex by cylinder were obtained. The form of these functions argues to chaotic behavior of the scattering which means that an additional analytical integral is absent.

An autonomous systems with quasiperiodic dynamics: examples and properties

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A traditional examples of a systems with quasiperiodic oscillations are non-autonomous systems. However, autonomous quasiperiodicity is possible in some generators of quasiperiodic oscillations. One of the example of this kind system is well-known Chua circuit [1]. Another example, is one of modification of the Lorenz system so called Lorenz-84 [2,3]. For this system invariant torus occurs on base of cycle of period 2. Prof. Anishchenko with coauthors suggest four-dimensional system in form of modification of generator of Anishenko-Astakhov [4]. This system also demonstrates bifurcation of torus doubling. In more detail is considered a new model of three-dimensional autonomous generator based on oscillator with hard excitation, which was suggested by prof. Kuznetsov, and its modification [5-7]. A chart of dynamical regimes and of Lyapunov exponents for autonomous system is presented, the possibility of realizing of hidden-attractors and main scenarios of destroying tori is discussed. A picture of synchronization for this system with external action is considered, in particular, Arnold resonance web was revealed in this system [8]. A structure of parameter plane for coupled generator of quasiperiodic oscillations is discussed. Results of experimental studying is presented.

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References

1. Matsumoto T. // TIIER. 1987.Vol.75, No.8. PP.66-87. (in Russia)
2. Lorenz E.N. // Tellus. 1984. Vol. 36A. PP. 98-110.
3. Shil'Nikov A., Nicolis G., Nicolis C. // Int. J. Bifurcation Chaos. 1995. Vol. 5, No.6. PP. 1701-1711.
4. Anishchenko V.S., Nikolaev S.M., Kurths J. // Phys. Rev. E. 2007. Vol. 76. P. 046216.
5. Kuznetsov A.P., Kuznetsov S.P., Stankevich N.V. // Commun Non Sci Num Simul. 2010. Vol. 15. PP. 1676-1681.
6. Kuznetsov A.P., Kuznetsov S.P., Mosekilde E. and Stankevich N.V. // J. Phys. A. 2015. Vol.48. No.12. P. 125101
7. Kuznetsov A.P., Kuznetsov S.P., Mosekilde E. and Stankevich N.V. // EPJ ST. 2013. 222. No.10. P.2391-2398.
8. Stankevich N.V., Kurths J., Kuznetsov A.P. // Commun Non Sci Num Simul. 2015. Vol. 20. No 1. PP. 316-323.

Invariant curves of quasi-periodically forced maps

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This talk deals with invariant curves of quasiperiodically forced smooth maps depending on a parameter,

$$\left.\begin{array}{rcl} \tilde{x} &=& f(x,\theta,\mu), \\ \tilde{\theta} &=& \theta+\omega \end{array}\right\}.$$

where $x \in R^n$, $\theta \in T$, ω is Diophantine and μ is a real parameter. The map f is assumed to be of class C^r, $r \geq 1$. An invariant curve is a C^1 map $\theta \mapsto x(\theta)$ such that $f(x(\theta),\theta,\mu) = x(\theta+\omega)$. Suppose that for one value of the parameter $\mu = \mu_0$, we have an attracting invariant curve of the system. We are interested in the behaviour of this curve when its Lyapunov exponent tends to zero when the parameter tends to some critical value μ_1. In particular, we want to study the fractalization phenomenon that might give rise to the appearence of Strange Non-Chaotic Attractors. In order to do this, we will focus on the most simple non-trivial situation, quasiperiodically forced affine systems in the plane. We will show that these systems have invariant curves that displays a fractalization process when the parameter tends to a critical value, if the corresponding linear system is non-reducible. This is a joint work with A. Jorba, N. Fagella and M. Jorba-Cusco.

On Bonatti-Diaz cycles

D. Turaev (Imperial Colledge, London)

Piecewise Contractions as Models of Regulatory Networks

E.Ugalde

Synchronization of traveling waves in the active medium with periodic boundary conditions

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The model of a one-dimensional active medium, which cells are the FitzHugh-Nagumo oscillators, is studied with periodical boundary conditions. Such medium has three different regimes in dependence on the parameters values. These regimes correspond to the self-sustained oscillations, excitable dynamics or bistability of the medium cells. Periodic boundary conditions provide the existence of traveling wave modes in all mention cases without any deterministic or stochastic excitation. The local and distributed priodic force influence on the medium are studied. The phenomena of the traveling wafrequency locking are found in all three regimes of active medium. The comparison synchronization effects in self-oscillatory, excitable and bistable regimes of the activmedium is fulflled.

[1] Y. Kuramoto, D. Battogtokh, Nonlinear Phenom. Complex Syst. 5, 380 (2002).

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Periodic forcing of a 2-dof Hamiltonian undergoing a Hamiltonian-Hopf bifurcation

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We consider a 2-dof Hamiltonian undergoing a Hamiltonian-Hopf bifurcation. We will review several aspects of this bifurcation. Our goal is to study the effect of a periodic perturbation acting on the previous Hamiltonian. Particular interest will be focused on the behaviour of the splitting of the invariant manifolds of the complex-saddle. Several aspects (normal forms, Poincaré maps, existence of invariant tori, spliting of the invariant manifolds, chaos, return maps,...) will be analysed as a combination of theoretical and numerical tools.
This is part of an ongoing work with E. Fontich and C. Simo.

New invariants of topological conjugacy of non-invertible inner mappings

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Let $f : X \to X$ be an inner surjective map of a locally compact locally connected metric space X. Recall that an inner map is an open and isolated map. A map is open if the image of an open set is open. A map is isolated if the pre-image of a point consists of isolated points.

The papers [2, 3]introduced a set of new invariants of topological conjugacy of non-invertible inner mappings that are modeled from the invariant sets of dynamical systems generated by homeomorphisms. Those new invariants are based on the analogy between the trajectories of a homeomorphism and the directions in the set of points having common image which is viewed as having 2 dimensions.

In particular, this papers introduced the sets of neutrally recurrent and the neutrally non-wandering points related to the dynamics of points and neighborhoods in that "extra" dimension. Those invariants provide a natural language for the topological classification of many classes of polynomial maps and also allow to define analogs of many well known classes of invertible maps such as Smale diffeomorphisms for the non-invertible inner maps.

[1] Yu. Yu. Trokhimchuk *Differentiation, inner mappings and criteria of analyticity*.- Inst. Math. NAN of Ukraine, 2008.

[2] I. Yu. Vlasenko *Dinamics of inner mappings*.- Ukr. Nonlinear Oscillations, 2011, v.14, No.2, 181-186.

[3] I. Yu. Vlasenko *Inner mappings: topological invariants and their applications*.- Inst. Math. NAN of Ukraine, 2014.

О законах сохранения в гидродинамике

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В [1] была предложена новая квантово-гидродинамическая аналогия и на ее основе найдены новые точечные законы сохранения для баротропных движений идеальной сжимаемой жидкости. Квантово-гидродинамическая аналогия позволяет также установить ряд сохраняющихся величин в форме специфических для квантовой механики средних от операторов.

В представлении Лэкса уравнений гидродинамики идеальной несжимаемой жидкости оператор $\hat{A} = -i(\boldsymbol{v} \cdot \nabla)$ самосопряженный. Если его отождествить с гамильтонианом, то аналогом квантовомеханического состояния ψ будет функция лагранжевых координат *ψ(a,b,c)* [1]. При этом оператор $\hat{L} = -i(\mathrm{rot}\boldsymbol{v} \cdot \nabla)$ сохраняющийся и, следовательно,

$$\int {}^*\hat{L}\, dV = \text{const.}$$

Поскольку оператор $\hat{L}$ переводит функцию лагранжевых координат в функцию тех же координат и самосопряженный, то сохраняющимися величинами являются также средние от нечетных степеней оператора $\hat{L}$:

$$\int {}^*\hat{L}^{2n+1}\, dV = \text{const}; n = \overline{0, \infty}.$$

Те же соображения, но примененные к $\int {}^* dV = \text{const}$ (сохранение нормы) дают сохраняющиеся средние от четных степеней оператора $\hat{L}$:

$$\int {}^*\hat{L}^{2n}\, dV = \text{const}, n = \overline{0, \infty}.$$

Таким образом, среднее от любой степени оператора $\hat{L}$ сохраняется. Здесь усреднение проводится с любой функцией лагранжевых координат , достаточно гладкой и достаточно быстро убывающей на бесконечности.

Можно показать, что в случае баротропных движений идеальной сжимаемой жидкости имеют место аналогичные законы сохранения, но в метрике с весом ρ, то есть

$$\int {}^*\left(\frac{rot\ \boldsymbol{v}}{\rho} \cdot \nabla\right)^n \rho dV = \text{const};\ n = \overline{0, \infty}.$$

Литература

1. Дерендяев Н. В. Квантово-гидродинамическая аналогия. Доклады АН, 2014, Т.457, №2, с. 167-169.

Kinetic of hydrogen absorption by palladium near saturation limit

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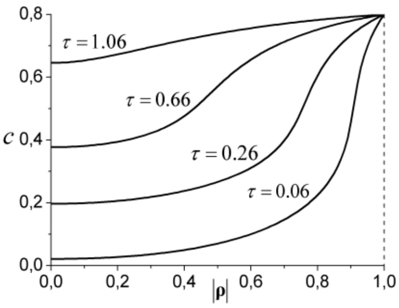
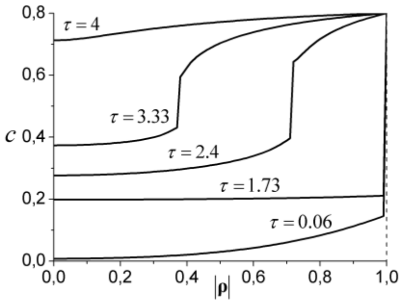
Hydrogen is a common and energy-intensive material. There is hydrogen in organic molecules, fuel cells, nuclear reactors etc.

Hydrogen in metals is using in accumulators, hydrogen filtration etc. It is known that hydrogen in the metal exists in α and β phases. There is critical temperature, when hydrogen in β phase isn't exists. In this work was tried to describe hydrogen absorption kinetics with temperature a few higher then critical by taking into account hydrogen-hydrogen interaction in metals. For the calculation was introduced constant, indicating the attraction of hydrogen atoms to each other inside the metal. The attraction was considered in the nearest neighbors' model. From these approximations were eventually a corresponding diffusion equation.

$$\frac{\partial c}{\partial t} = \nabla\left\{\left[1 - \frac{4c(1-c)}{\theta}\right]\nabla c\right\},$$

where τ, θ, ∇ - dimensionless time, temperature and gradient respectively. It was also made the numerical solution of the diffusion equation for a spherical palladium sample placed in hydrogen atmosphere. Result are on figures.





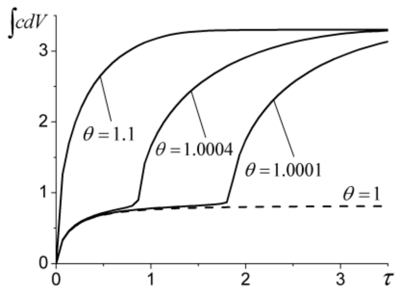


Fig. 1 **Fig. 2** **Fig. 3**

Fig.1. Temporal evolution of radical distribution of absorbed hydrogen concentration in a spherical monocrystalline sample with temperature value θ = 1.1.

Fig.2. The same as in Fig.1 for θ = 1.0001.

Fig.3. Cumulative absorption curves for the temperature values θ = 1.0001, 1.0004 and 1.1.

[1] G. Alefeld, I. Felkel, Hydrogen in metals, Mir, Moskow (1981).